Problem 14

In this problem, we have a circular industrials disk sander with radius one meter. And it has a moment of inertia about g of 0.5 kilograms meters squared. It is rotating at a speed of 100 .. and 1100 RPM, just after it's been disconnected from the power source. At this time, a block of wood is pushed against the sender with a constant force at a distance of five centimeters from the center of the sender G. So that this disk slows down. This takes 22nd to come to a complete stop. And we are asked what is the force that is required to stop this disc in exactly 20 seconds. There are no friction loss, other frictional losses on the disk other than this block that we are applying to slow it down. And we can take the coefficient of friction to be 0.8. Again, between the block and the disk. So this is a clearly an angular momentum problem. And we're going to look at the two different states, right. So we start at a state where we have this disk spinning with omega, omega one, and that is the initial state, we're given this omega over here, that's you can derive it from 1100 rpm. And in that state is is defined. Now, what happens is, we get to a final state state two where this angular momentum, or this angular velocity slows down until zero. So at the final state state two omega equals to zero, which also means we will have no angular momentum, right? Because angular momentum is i omega, and we have no angular velocity. So what happens in between those two states? Well, what we're doing is we are applying an angular impulse, right, we are applying a force over time. And this is slowing down the disk. Now what is slowing down the disk, we're applying a force in the direction of into the page right in the direction of that piece of wood. But what's actually slowing down the disk is a force of friction, that points equal and opposite to the direction of motion. So this is the force of friction that is generated from the normal force or the contact force between this brick of wood and the disk. So let's start writing that down. And first write a freebody diagram to see what is happening. So we have G at the center of the disk, this is the center of gravity. And we have omega, the angular velocity of the disk with which the disk is spinning right. At this point here, we'll call this point of contact point P, which is located at a distance of d from the center of gravity. This point here is where we the center of where we apply the brick, or the block of wood, right. And this point is where we apply essentially, where that load is applied. So where the force F is applied. Now I've drawn it dotted because this force goes into the page here. So you can also look at it as essentially a cross write into the page, right? Because our coordinate system, x to the right, y positive, and then z is going to be out of the page with a positive rotation that way. And this is z out of the page. So we know that at point P, we have force F. Now this is not exactly force F, this is force F due to the normal force, right? So when we apply this block of wood, we're actually applying a force with our hand on the end of the block of wood on this end, right on this face. What's happening is this block of wood is then touches the disk and a normal force equal and opposite to f is generated onto this block of wood to balance it right. And to balance this force on the block of wood there is going to be an opposite force of F that points in the same direction as F and has the same magnitude of F, but onto the disk. So this is due to that normal force. Now this force is acting a distance at a distance d away from the center of the disk, so the center of gravity and about which this disk is spinning. So this will generate a moment, and this will come in later in a second with our angular impulse. But why is this force F generating a moment because this force F is not itself is not, it's generating a moment about the y axis, right? It is spinning everything in the y axis in this direction. But we're not interested in that moment, we're actually interested in the moment caused by the force of friction due to that force, right. So, we're the force of friction, in this case, points upwards, because it is going to be opposite to the direction of the velocity. So if we have the angular velocity pointing in the clockwise direction, we are going to have a velocity at point P, which I'll just draw a bit offset that points downwards. So this is VP, right? I drew this offset just to declutter the diagram. But again, this vector would be located over here, right velocity points down, so we have our force of friction pointing in the opposite direction. That means it points up. And this force of friction, this is the force of friction that is causing a moment in the x, y plane or around z, right? This moment is twisting everything about the z axis like that. So why is this important? Well, because we know if we look at angular momentum and conservation of angular momentum, we know that we're starting from an angular momentum of H, G, one, right? This is when we have an initial velocity, and initial angular velocity plus delta H, G. And this is from one to two is equal to h, g two. And in this special case, we can cancel out HG two, because we know that the angular velocity is zero at our final point, and therefore there is no angular momentum. Right. Now, what is this term over here? Well, this term over here is defined as delta H. G 1 to 2, is equal to the sum of the integral from t one to t two of the moment in dt. And why do we have a sum here? Well, this is because if there were multiple moments, we'd have to sum up all the contributions from all the moments. In this case, there's only one so we can eliminate this sum over here, and just take care of the one moment. Now what is this? This is the angular impulse, right? This is the integral of the moment in time. And in this case, this moment will be constant, because this force, this force of friction here, and this radius here, are not changing in time. So the moment will be continent time. And so this is essentially the moment multiplied by the time that this moment is acting, right. So this is the angular impulse. So the change in angular momentum is from state one to state two is equal to that angular impulse due to that moment, right. So given these equations, let's now find the values for h G one and delta H, G, one to two, and then as a function of the applied force F and then solve for this applied force F assuming conservation of angular momentum. So looking at our freebody diagram, we know that this force of friction, f of f is equal to mu k times f, right? That force normal force F. And so we can we know have the force of friction as a function of our kinetic friction coefficient, which is given and the value of f that we're trying to solve for. The next thing to do is to define each right so what is H G? H G is equal to I G, omega, right and this would be a vector equation. And in this case, we can get rid of the vector, because we're going to solve everything in terms of scalars. But what this what we need to find here is this IG, right? Because in the initial state, we know the angular velocity, we need to find the mass moment of inertia about G. So about the point where everything is spinning. So let's go ahead and do that. So I g is equal to 0.5 kilograms meters squared, because that is given in the question. All right, the last thing we need to figure out is omega. And omega, we're also given in the question, but we're given omega in rpm, and we need to convert this into radians per second. So omega one is equal to two pi r, times 1100 RPM divided by 60. And this is going to be equal to 115.2 radians per second. We're also given in the question that omega two is equal to zero. And that's why we made the assumption that H G two is equal to zero, right? So now we can, we can find the initial angular momentum. So H G one is equal to 0.5 kilograms, meters squared, times 115.2 radians per second. This is equal to 57.6 kilograms, meters squared per second. And this is going to be our value for the initial angular momentum. Now let's try and solve for this term, the angular impulse. So we know that we can get rid of that sum. So the integral from t one to t two, and we know T one and T two T one, we're going to set up the zero time step, and T two is going to be the 22nd time step, because that is is given in the question, we're applying this moment for 20 seconds of the moment in dt is equal to so we can pull this moment out, right? We can pull it out of the integral because we said it's constant. So the moment from T one equals to zero, t two equals to 20 seconds in dt, right? Because we said that this moment does not change with time. So what we can do now is just multiply the moment by 20 seconds, times the time, right, the delta t. So this integral here just becomes that 20second. Right? So that's why I have m times 20 seconds, just to keep the unit's consistent, this is equal to that. So now we need to find this moment. What is the moment? Well, m is equal to r cross F. Right? This is the general definition, what is the specific moment in this case, this is r of P with respect to g, crossed to that force of friction, f of f, right, so it's going to be the cross product between this force over here and drive and read this radius over here, r of P with respect to G. right, so we can clearly see that these forces are 90 degrees apart. So there's a 90 degree angle over here. And so we can see that this force, or this radius cross to this force over here, is going to give a vector that points into or out of the page to get the third component to be perpendicular. And this so this moment, will be turning around z either clockwise or anti clockwise, but it's going to turn around z. And so we can turn this factorial equation over here into just a scalar equation by ignoring the vectors severything will be at 90 degrees, and we have confirmed that anything is rotating about this z axis, which is what we're doing with the sum of angular momentum about, right. So we can turn this into m being equal to r, which is just going to be D, right? Because this r of P with respect to g is just this distance d. And we said we're just multiplying the magnitudes times the force of friction, which we said is mu k times f. And then we have that we can take this, plug it into there, and get that the delta H G from one to two is equal to d, mu k, f times 20 seconds. And so we've now come up with a definition for this term over here. Okay, so now we can put everything together into this master equation over here. So we're going to equate this term with this term, and solve for the force F. So let's go ahead and do that. We have that 57.6 kilograms, meters squared per second, is equal to 0.05 meters, times 0.8, which is our friction coefficient times f times 20 seconds, and therefore, f is equal to 72 Newtons. And in the vectorial form, f is equal to 72 Newtons in the negative k hat direction. And this is our final answer for the problem.